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FOREIGN TECHNOLOGY DIVISION

MOSCOW. INSTITUTE OF ENGINEERING PHYSICS. USE OF QUANTUM FIELD THEORY METHODS IN THE PROBLEM OF MANY BODIES (SELECTED ARTICLES)

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ABSTRACT: Each of the ensuing paragraphs represent one article's abstract:

IN A PLASMA, BY A.I. ALEKSEYEV

General formulas are derived for the energy loss of a charged particle passing with any velocity through a multicomponent plasma of arbitrary temperature;

PLASMA, BY M.A. TROITSKIY

Because of the possibility that new statistical-physics methods can give more accurate results on the bremsstrahlung of a high temperature plasma and on recombination and line-spectrum radiation of low-temperature plasma, quantum field theory methods of statistical physics are used to determine the spectral expansion of bremsstrahlung intensity per unit volume of a plasma, with an allowance made for the shielding of the Coulomb yield of ions.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

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Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
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К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
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О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

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FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

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The Application of Quantum-Field-Theory Methods to Multibody Problems. Moscow, Gosatomizdat, 1963
Pages: 65-82

DECELERATION OF A RELATIVISTIC PARTICLE IN A PLASMA

A. I. Alekseyev

By the method of Green's functions there are found general formulas for energy losses of a charged particle passing with any speed through a multicomponent plasma of arbitrary temperature. In detail there is investigated region of nonrelativistic temperature of the plasma and arbitrary speed of flying particle. The work was completed in 1960.

Usually in calculating energy losses of a charged particle passing through plasma, losses connected with pair collisions, and losses caused by excitation of plasma waves are examined separately. Both named parts of energy losses have an identical order of magnitude. Because of the incorrectness of the methods used, the final results of total losses, obtained by different authors, as a rule, differ by numerical factor under the sign of logarithm. In connection with this it seems natural to apply to the given phenomenon new methods with use of Green's functions and diagram technique which makes it possible more clearly and correctly to solve the posed problem. The method of Green's functions together with diagram technique in application to energy losses of particle in a plasma were first worked out by Larkin [1] who, however, from the very beginning was limited to the nonrelativistic region. Below there is proposed a further development of the indicated method, in reference to the case of

arbitrary velocities of particles passing through a multicomponent plasma (see also [2]). Furthermore, in the present discussion the method is readily extended to other problems: bremsstrahlung and the production of pairs during passage of charged particles through a plasma, radiation of plasma, deceleration of charged particles in matter, etc.

Probability of Scattering of a Particle in a Plasma

We shall examine the statistical system in a thermal equilibrium consisting of several levels of fermions interacting with each other by means of an electromagnetic field (multicomponent plasma). For generality we shall assume the problem relativistic. Such a system in a nonrelativistic approximation may describe, for example, an electron-ion plasma containing several levels of ions. For calculating the stopping power of multicomponent plasma we shall write, in Schrödinger notation, the Hamiltonian of the plasma and of the external flying particle, interacting by means of an electromagnetic field*

$$H = H_0 + H_0' + H_1', \quad H_0 = \sum_{\lambda} H_{\lambda} + H_{\gamma} + H_{e1}, \tag{1}$$

$$H_{\lambda} = \sum_{p} \varepsilon_{p}^{\lambda} \Big(\sum_{r=1}^{2} a_{pr}^{\lambda +} a_{pr}^{\lambda} + \sum_{r=3}^{4} b_{pr}^{\lambda +} b_{pr}^{\lambda} \Big), \tag{2}$$

$$H_{\gamma} = \sum_{k, j=1}^{2} \omega_{k} c_{kj}^{+} c_{kj}, \quad H_{e1} = \int j(x) A(x) d^3x, \tag{3}$$

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*Everywhere the system of units is used in which $\hbar = c = 1$ and there is adopted the following rule of summation over the vector indices: $pq = p_{\nu}q_{\nu} = p_4q_4 - p_1q_1 - p_2q_2 - p_3q_3$. Here $\hat{p} = p_{\nu}\gamma_{\nu}$, where γ_4 and $\gamma_{1,2,3} = \gamma_4\alpha_{1,2,3}$ are ordinary Dirac matrices.

$$j_\nu(x) = \sum_\lambda e_\lambda N[\bar\psi_\lambda(x)\gamma_\nu \psi_\lambda(x)] = \sum_\lambda \frac{e_\lambda}{2}\gamma_{\nu\alpha\beta}\times$$
$$\times[\bar\psi_{\lambda\alpha}(x)\psi_{\lambda\beta}(x) - \psi_{\lambda\beta}(x)\bar\psi_{\lambda\alpha}(x)]. \tag{4}$$

$$\psi_\lambda(x) = V^{-1/2}\Big(\sum_{p,r=1}^{2} a^\lambda_{p\,r}\, u^r_\lambda(p)\, e^{ipx} + \sum_{p,r=3}^{4} b^{\lambda+}_{p\,r}\, u^r_\lambda(-p)\, e^{-ipx}\Big) \tag{5}$$

$$\bar\psi_\lambda(x) = V^{-1/2}\Big(\sum_{p,r=1}^{2} a^{\lambda+}_{p\,r}\, \bar u^r_\lambda(p)\, e^{-ipx} + \sum_{p,r=3}^{4} b^\lambda_{p\,r}\, \bar u^r_\lambda(-p)\, e^{ipx}\Big), \tag{6}$$

$$A_\nu(x) = \sum_{k,j=1}^{4} (2\pi/\omega_k V)^{1/2}(c_{kj}e^{ikx} + c^+_{kj}e^{-ikx})\, l^j_\nu, \tag{7}$$

where $H\lambda$ and $H\gamma$ are Hamiltonians of the free fermion level λ and of the photon fields, respectively, and $H_{\Pi 1}$ is their interaction operator. H_Π is the Hamiltonian of the plasma. H_0' is the Hamiltonian of free field of flying particle (fermion), and H_1' is the operator of its interaction with photon field. H_0' and H_1' have the same structure as H_λ and $H_{\Pi 1}$ in which the subscript λ is discarded. At $r = 1.2$, $u^r_\lambda(\mathbf{p})$ is the solution of the Dirac equation

$$(\hat p - m_\lambda)\, u^r_\lambda(\mathbf{p}) = 0 \tag{8}$$

for positive energy $\varepsilon^\lambda_p = \sqrt{\mathbf{p}^2 + m^2_\lambda}$, and at $r = 3.4$ – for a negative energy, equal to $\sqrt{\mathbf{p}^2 + m^2_\lambda}$. Then, $u = u^*\gamma_4 aA(\mathbf{x})$ is the four-dimensional vector potential of electromagnetic field; $a^\lambda_{\mathbf{p}}(b^\lambda_{\mathbf{p}\,r})$ and $a^\lambda_{\mathbf{p}\,r}{}^+(b^\lambda_{\mathbf{p}\,r}{}^+)$ are operators respectively of the absorption and creation of a fermion (anti-fermion) of sort λ with momentum $\mathbf{p}$, polarization r, and energy $\varepsilon^\lambda_{\mathbf{p}}$; and $c_{\mathbf{k}\,j}$ and $c^+_{\mathbf{k}\,j}$ are analogous operators of photon with momentum $\mathbf{k}$, polarization l^j and energy $\omega_{\mathbf{k}} = |\mathbf{k}|$. The sign

N before the operators designates the N-product [3], V is the volume of plasma, and e_λ is the charge of fermion of level λ. The S-matrix describing quantum-mechanical transitions of plasma and flying particle, satisfies the equation

$$i\frac{\partial S}{\partial t}=(H_a+H_0'+H_1')S. \tag{9}$$

We now use the transformation

$$S=e^{-i(H_a+H_0')t}s \tag{10}$$

and then turn to another representation, in which the operators of field $\psi'(x)$ of a flying particle are written in the interaction picture

$$\psi'(x)=e^{iH_0't}\,\psi'(x)e^{-iH_0't}, \tag{11}$$

and operators of fermion $\psi_\lambda(x)$, $\overline{\psi}_\lambda(x)$ and photon A(x) field – in the Heisenberg picture

$$\begin{aligned}\psi_\lambda(x)&=e^{iH_at}\,\psi_\lambda(x)e^{-iH_at},\\ A(x)&=e^{iH_at}\,A(x)e^{-iH_at}.\end{aligned} \tag{12}$$

Here the operator A(x) in case of a Lorenz gauge transformation satisfies the equation

$$\left(\nabla^2-\frac{\partial^2}{\partial t^2}\right)A(x)=-4\pi\sum_\lambda e_\lambda\,N[\overline{\psi}_\lambda(x)\gamma\psi_\lambda(x)] \tag{13}$$

with a solution of the type

$$\begin{aligned}A_\mu(x)=A^0_\mu(x)+\sum_\lambda e_\lambda\int D^F_{0\mu\nu}(x-x')N\times\\ \times(\overline{\psi}_\lambda(x')\gamma_\nu\psi_\lambda(x'))d^4x',\end{aligned} \tag{14}$$

where $A^0_\mu(x)$ is the free photon field, and $D^F_{0\mu\nu}(x-x')$ is the function of propagation of photon in quantum electrodynamics in the zero approximation

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) D^F_{0\mu\nu}(x) = -4\pi \delta_{\mu\nu} \delta(x), \tag{15}$$
$$\delta_{44} = 1, \ \delta_{11} = \delta_{22} = \delta_{33} = -1.$$

With the other gauge-transformation of the potentials A(x), equations (13), (15) and function $D^F_{0\mu\nu}(x - x')$ vary; however, general form of the solution (14) will be maintained with arbitrary guage-transformation potentials. On the right-hand side of equation (13) is the sum of the currents formed by each level of fermions of the plasma.

The transformation (10-12) in accordance with the sense of the posed problem assumes that the interaction of a flying particle with plasma particles is engaged and is disengaged at moments of time $t = -\infty$ and $t = +\infty$, respectively, whereas interaction between particles of plasma one with another remains continuously engaged $e_\lambda \neq 0$.

In the indicated representation the s-matrix is determined in the following way:

$$i \frac{\partial s}{\partial t} = H_1' s, \quad s = T e^{-i\int H_1'(x) d^4x}, \tag{16}$$
$$H_1'(x) = e' N(\bar{\psi}'(x) \hat{A}(x) \psi'(x)),$$

where e' is the charge of flying particle; the operator A(x) is given by expression (14) and the symbol T before operators indicates a T-product [3]. The s matrix (16) describes scattering of flying particles in a multicomponent plasma as a single whole. To an equal degree the s-matrix (16) is applicable for the description of the phenomenon of scattering with radiation, the formation of pairs, etc.

We shall then assume a flying particle, moving with a speed v, sufficiently rapid $e_\lambda e'/\hbar v \ll 1$, in order that its interaction with electromagnetic field created by the plasma may be examined according

to the theory of perturbations. Then the matrix element of the s-matrix describing the scattering of an external particle with its transition from a state with momentum **p** and polarization r to a state with $\mathbf{p}'$ and r', and the plasmas – from a state n to a state m, is determined by following formula:

$$\left.\begin{aligned} s_{m\mathbf{p}'r',\,n\mathbf{p}r} &= -i\sum_{\lambda} e' e_{\lambda} \left(m\mathbf{p}'r' \,\Big|\, \int \bar{\psi}'(x)\gamma_{\mu}\psi'(x) \times \right. \\ &\times D^{F}_{0\mu\nu}(x-x')\bar{\psi}_{\lambda}(x')\gamma_{\nu}\psi_{\lambda}(x')\, d^4x\, d^4x' \,\big|\, n\mathbf{p}r\big) = \\ &= -\frac{i(2\pi)^4 e'}{V}\left(\bar{u}^{r'}_{\mathbf{p}'}\gamma_{\mu} u^{r}_{\mathbf{p}}\right) D^{F}_{0\mu\nu}(\mathbf{q},\omega)\sum_{\lambda} e_{\lambda} \times \\ &\times \left(\bar{\psi}_{\lambda}(0)\gamma_{\nu}\psi_{\lambda}(0)\right)_{mn}\delta(\mathbf{q}-\mathbf{p}_{mn})\,\delta(\omega-\omega_{mn}); \\ \mathbf{q} &= \mathbf{p}-\mathbf{p}'; \quad \omega = \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{q}}; \quad \mathbf{p}_{mn} = \mathbf{P}_m - \mathbf{P}_n; \\ \omega_{mn} &= E_m - E_n; \quad \varepsilon_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + M^2}, \end{aligned}\right\} \tag{17}$$

where the subscripts n and m denote the state of plasma, in which the total energy E, the difference between total number of fermions and anti-fermions $N = \sum_{\lambda} N_{\lambda}$, and also the total momentum **P** of the plasma have specific values; **q** and ω are, respectively, the momentum and energy transmitted to the plasma during scattering of flying particle of mass M and of charge e'.

The probability dW of the scattering averaged on the basis of the initial and integrated on the basis of the final spin states of a flying particle, and also integrated over all final states of plasma and statistically averaged on the basis of initial states of plasma by means of the Gibbs distribution

$$e^{\left(\Omega + \sum_{\lambda}\mu_{\lambda}N_{\lambda} - E_n\right)\beta}, \tag{18}$$

where $\beta = 1/kT$; Ω is the thermodynamic potential of a plasma; μ is the chemical potential of fermions of sort λ, and N_{λ} the difference between total number of fermions and anti-fermions of sort λ, is

determined in the following way:

$$dW = \frac{e'^2}{(2\pi)^3} T_{\mu\mu'} D^{P}_{0\,\mu'\nu'}(\mathbf{q}, \omega) D^{P}_{0\,\mu\nu}(\mathbf{q}, \omega) \Phi_{\nu\nu'}(\mathbf{q}, \omega) d^3p', \tag{19}$$

$$T_{\mu\mu'} = \frac{1}{2\varepsilon_p \varepsilon_{p'}} [p_\mu p'_{\mu'} + p'_\mu p_{\mu'} - \delta_{\mu\mu'}(pp' - M^2)], \tag{20}$$

$$\Phi_{\nu\nu'}(\mathbf{q}, \omega) = (2\pi)^3 \sum_{m,n} e^{(\Omega + \sum_{\lambda''} \mu_{\lambda''} N_{\lambda''} - E_n)\beta} \sum_{\lambda, \lambda'} e_\lambda e_{\lambda'} \times$$

$$\times (\bar{\psi}_\lambda(0) \gamma_\nu \psi_\lambda(0))_{nm} (\bar{\psi}_{\lambda'}(0) \gamma_{\nu'} \psi_{\lambda'}(0))_{mn} \delta(\mathbf{q} - \mathbf{p}) \delta(\omega - \omega_{mn}), \tag{21}$$

where $\Phi_{\nu\nu'}(\mathbf{q}, \omega)$ is expressed in terms of the correlation function, which in turn is intimately connected with Green's thermodynamic functions of particles of a plasma, p_μ is the four-dimensional momentum vector of a flying particle, in which $p_4 = \varepsilon_p$.

The Correlation Function

We shall examine the following correlation function:

$$K_{\mu\nu}(x_1 - x_2) = \sum_{\lambda, \lambda'} e_\lambda e_{\lambda'} Sp\{e^{(\Omega + \sum_{\lambda''} \mu_{\lambda''} N_{\lambda''} - H_n)\beta} \times$$

$$\times T_\tau [(\tilde{\bar{\psi}}_\lambda(x_1) \gamma_\mu \tilde{\psi}_\lambda(x_1)) \cdot (\tilde{\bar{\psi}}_{\lambda'}(x_2) \gamma_\nu \tilde{\psi}_{\lambda'}(x_2))]\}, \tag{22}$$

in which x is the totality of $\mathbf{x}$ and of the variable τ, varying within the limits $0 \leq \tau \leq \beta$. The symbol T_τ designates the T-product [3], in which the order of operators proceeds according to the variable τ, and the tilde sign $\sim$ denotes operators in the "Heisenberg picture," for example

$$\tilde{\psi}_\lambda(x) = e^{-(\sum_\lambda \mu_\lambda N_\lambda - H_n)\tau} \psi_\lambda(\mathbf{x}) e^{(\sum_\lambda \mu_\lambda N_\lambda - H_n)\tau} \tag{23}$$

In order to ascertain the indicated relationships between functions (21) and (22) we shall make, by following Landau's [4] method, a spectral expansion of the function $K_{\mu\nu}(x)$ (22). We shall obtain

$$K_{\mu\nu}(q, \tau) = \begin{cases} \int\limits_{-\infty}^{\infty} \Phi_{\mu\nu}(q, \omega) e^{-\omega\tau} d\omega, & \tau > 0; \\ \int\limits_{-\infty}^{\infty} \Phi_{\mu\nu}(q, \omega) e^{-\omega(\beta+\tau)} d\omega, & \tau < 0; \end{cases} \tag{24}$$

where $-\beta \leq \tau \leq \beta$, and the function $\Phi_{\mu\nu}(q, \omega)$ is determined by expression (21). We shall periodically extend function (24) to the entire axis τ, then at any τ there will be fulfilled the relationship

$$K_{\mu\nu}(q, \tau) = K_{\mu\nu}(q, \tau + \beta). \tag{25}$$

Then, in completing the Fourier transform

$$K(x, \tau) = \frac{1}{(2\pi)^3 \beta} \sum_{\omega_n} \int K(q, \omega_n) e^{i(qx - \omega_n \tau)} d^3q, \tag{26}$$

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we shall obtain, by taking into account relationship (25), 71

$$K_{\mu\nu}(q, \omega_n) = \int\limits_{-\infty}^{\infty} \frac{\Phi_{\mu\nu}(q, \omega)(1 - e^{-\omega\beta})}{\omega - i\omega_n} d\omega, \tag{27}$$

$$\omega_n = 2n\pi/\beta, \quad n = 0, \pm 1, \pm 2 \ldots$$

Integral (27), considered formally as function of variable $i\omega_n$, determines analytic function $K^a_{\mu\nu}(q, \omega)$ in the upper half-plane

$$K^a_{\mu\nu}(q, \omega) = \int\limits_{0} \frac{\Phi_{\mu\nu}(q, \omega')(1 - e^{-\omega'\beta})}{\omega' - \omega} d\omega' =$$

$$= P \int \frac{\Phi_{\mu\nu}(q, \omega')(1 - e^{-\omega'\beta})}{\omega' - \omega} d\omega' + i\pi\Phi_{\mu\nu}(q, \omega)(1 - e^{-\omega\beta}), \tag{28}$$

which coincides with $K_{\mu\nu}(q, -i(i\omega_n))$ at an infinite set of points $\omega = i\omega_n$ ($\omega_n > 0$), having a point of bunching. By the theorem of analytic continuation we conclude that $K^a_{\mu\nu}(q, \omega)$ is analytic continuation of function $K_{\mu\nu}(q, -i(i\omega_n))$ (27) on upper half-plane of complex variable ω

$$K^a_{\mu\nu}(q, \omega) = K_{\mu\nu}(q, -i\omega). \tag{29}$$

Inasmuch as $\Phi_{\mu\nu}(q, \omega)$ is the real function ω on real axis, then from formulas (28) and (29) we have

$$\Phi_{\mu\nu}(q, \omega) = \frac{\operatorname{Im} K_{\mu\nu}(q, -i\omega)}{\pi(1 - e^{-\omega\beta})}. \tag{30}$$

Thus, problem reduces to finding the function $K_{\mu\nu}(q, \omega_n)$, since the formal substitution of $\omega_n \to i\omega$ aids immediately in determining $\Phi_{\mu\nu}(q, \omega)$ by the formula (30). Calculations (see Appendix 1) result in the following general relationship:

$$K_{\mu\nu'}(q, \omega_n) D_{0\nu'\nu}(q, \omega_n) = -\Pi_{\mu\nu'}(q, \omega_n) D_{\nu'\nu}(q, \omega_n), \tag{31}$$

in which the polarization operator $\Pi_{\mu\nu}$ and Green's photon thermodynamic function $D_{\nu\nu'}$ are found from equations [2]

$$G_\lambda(p, \omega_m) = G_{0\lambda}(p, \omega_m) + G_{0\lambda}(p, \omega_m) M_\lambda(p, \omega_m) G_\lambda(p, \omega_m), \tag{32}$$

7 1 7 2

$$D_{\mu\nu}(k, \omega_n) = D_{0\mu\nu}(k, \omega_n) - D_{0\mu\mu'}(k, \omega_n)\Pi_{\mu'\nu'}(k, \omega_n) \times \times D_{\nu'\nu}(k, \omega_n) \tag{33}$$

$$\left.\begin{aligned} M_\lambda(p, \omega_m) &= \frac{e^2_\lambda}{(2\pi)^3\beta} \sum_{\omega_n} \int \gamma_\mu G_\lambda(p + k, \omega_m + \omega_n) \times \\ &\times \Gamma_\nu(p + k, \omega_m + \omega_n; k\, \omega_n) D_{\mu\nu}(k, \omega_n) d^3k. \\ \Pi_{\mu\nu}(k, \omega_n) &= \sum_\lambda \frac{e^2_\lambda}{(2\pi)^3\beta} \sum_{\omega_m} \int \operatorname{Sp} \gamma_\mu G_\lambda(p + k, \omega_m + \omega_n) \times \\ &\times \Gamma_\nu(p + k, \omega_m + \omega_n; p, \omega_m) G_\lambda(p, \omega_m) d^3p, \\ \Gamma(p, \omega_m; p', \omega'_m) &= \gamma + \Lambda(p, \omega_m; p'\, \omega'_m), \\ \omega_m = (2m + 1)\pi/\beta,\ \omega_n &= 2n\pi/\beta,\ m, n = 0, \pm 1, \pm 2, \ldots \end{aligned}\right| \tag{34}$$

Here Λ is determined in the form of series, being here the totality of all graphs of peak portion in addition to simple peak (point), and Green's thermodynamic functions of zero approximation $G_{0\lambda}$ and $D_{0\mu\nu}$, respectively, for fermion of level λ and photon are

equal to

$$G_{0\lambda}(\mathbf{p}, \omega_m) = [\mathbf{p}\boldsymbol{\gamma} - (i\omega_m + \mu_\lambda)\gamma_4 + m_\lambda]^{-1};$$
$$D_{0\mu\nu}(\mathbf{k}, \omega_n) = -\frac{4\pi\delta_{\mu\nu}}{\mathbf{k}^2 + \omega_n^2}. \tag{35}$$

Expression (35) corresponds to a Lorenz condition of the potentials (14). During another condition of the potentials (14) function $D_{0\mu\nu}$ has the form [5]:

a) in case $A_4 \equiv 0$*

$$D_{0ik}(\mathbf{k}, \omega_n) = \frac{4\pi}{\mathbf{k}^2 + \omega_n^2}\left(\delta_{ik} + \frac{k_i k_k}{\omega_n^2}\right);$$
$$D_{04i} = D_{0i4} = D_{044} = 0. \tag{36}$$

b) in case div $\mathbf{A} \equiv 0$

$$D_{0ik}(\mathbf{k}, \omega_n) = \frac{4\pi}{\mathbf{k}^2 + \omega_n^2}\left(\delta_{ik} - \frac{k_i k_k}{\mathbf{k}^2}\right). \tag{37}$$
$$D_{04i} = D_{0i4} = 0;\ D_{044}(\mathbf{k}, \omega_n) = -\frac{4\pi}{\mathbf{k}^2}.$$

In a concrete calculation of magnitudes $\Pi_{\mu\nu}(\mathbf{k}, \omega_n)$ and $M_\lambda(\mathbf{p}, \omega_m)$ in relativistic region there appear singularities associated with the renormalization of the charge and mass of plasma. In connection with this we shall rewrite equation (33) in such a form, which is the most convenient for investigating the question on the renormalization of the charge and mass of particles. For this purpose we note that there takes place the relationship [6]

$$k_l \Pi_{\mu l}(\mathbf{k}, \omega_n) - i\omega_n \Pi_{\mu 4}(\mathbf{k}, \omega_n) = 0. \tag{38}$$

In the most interesting case of the first approximation different from zero $\Pi^1_{\mu\nu}(\mathbf{k}, \omega_n)$ of the polarization operator $\Pi_{\mu\nu}(\mathbf{k}, \omega_n)$ (this

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*The vector designated by the Latin letters i, k, l, etc., runs through values from one to three (in distinction from the Greek letters ν, μ, etc., which take on all possible values 4, 1, 2, 3), where for δ_{ik} we have $\delta_{11} = \delta_{22} = \delta_{33} = 1$.

approximation is obtained if in the polarization operator (34) we replace all functions by their zero approximations) the relationship (38) is proven directly

$$k_l \Pi^1_{\mu l}(\mathbf{k}, \omega_n) - i\omega_n \Pi^1_{\mu 4}(\mathbf{k}, \omega_n) = \sum_\lambda \frac{e^2_\lambda}{(2\pi)^3 \beta} \times$$
$$\times \Big(\sum_{\omega_m} \int \mathrm{Sp}\, \gamma_\mu G_{0\lambda}(\mathbf{p}, \omega_m)\, d^3p - \sum_{\omega_m} \int \mathrm{Sp}\, \gamma_\mu G_{0\lambda} \times$$
$$\times (\mathbf{p} + \mathbf{k}, \omega_m + \omega_n)\, d^3p\Big) = 0, \tag{39}$$

where there has been used the equality

$$\mathbf{k}\gamma - i\omega_n\gamma_4 = [(\mathbf{p} + \mathbf{k})\gamma - (i\omega_m + i\omega_n + \mu_\lambda)\gamma_4 + m_\lambda] -$$
$$- [\mathbf{p}\gamma - (i\omega_m + \mu_\lambda)\gamma_4 + m_\lambda). \tag{40}$$

In the absence of a magnetic field the plasma is spatially an isotropic system and vector **k** is only vector on which the polarization operator (34) depends. Therefore, taking into account relationship (38) it is possible to write

$$\left.\begin{aligned} \Pi_{lk}(\mathbf{k}, \omega_n) &= \left(\delta_{lk} - \frac{k_l k_k}{\mathbf{k}^2}\right) \Pi^t(\mathbf{k}, \omega_n) + \frac{k_l k_k}{\mathbf{k}^2} \Pi^l(\mathbf{k}, \omega_n), \\ \Pi_{4l}(\mathbf{k}, \omega_n) &= \Pi_{l4}(\mathbf{k}, \omega_n) = -\frac{i k_l}{\omega_n} \Pi^l(\mathbf{k}, \omega_n), \\ \Pi_{44}(\mathbf{k}, \omega_n) &= -\frac{\mathbf{k}^2}{\omega^2_n} \Pi^l(\mathbf{k}, \omega_n), \end{aligned}\right\} \tag{41}$$

where the transverse Π^t and longitudinal Π^l parts of polarization operator (34) are determined as:

$$\Pi^t(\mathbf{k}, \omega_n) = \frac{1}{2}\left[\Pi_{ll}(\mathbf{k}, \omega_n) + \frac{\omega^2_n}{\mathbf{k}^2} \Pi_{44}(\mathbf{k}, \omega_n)\right], \tag{42}$$ 73

$$\Pi^l(\mathbf{k}, \omega_n) = \frac{k_l k_k}{\mathbf{k}^2} \Pi_{lk}(\mathbf{k}, \omega_n) = -\frac{\omega^2_n}{\mathbf{k}^2} \Pi_{44}(\mathbf{k}, \omega_n). \tag{43}$$ 74

Thus, the problem on removing the singularities in equation (33) reduces to removing the singularities in two scalar functions Π^t and Π^l, through which there is expressed the polarization operator $\Pi_{\mu\nu}$. Question about the removal of divergence in $\Pi_{\mu\nu}$ was investigated in works [7, 8].

Equation (33) has identical form independently of the conditions imposed on the potentials (14). In practical calculations the most convenient is the gauge transform of potentials in which

$$A_4 = 0. \tag{44}$$

In this case the solution of equation (33) has the form

$$D_{ik}(\mathbf{k}, \omega_n) = \left(\delta_{ik} - \frac{k_i k_k}{\mathbf{k}^2}\right) D^t(\mathbf{k}, \omega_n) + \frac{k_i k_k}{k^2} D^l(\mathbf{k}, \omega_n), \tag{45}$$
$$D_{4l} = D_{l4} = D_{44} = 0,$$

where

$$D^t(\mathbf{k}, \omega_n) = \frac{4\pi}{\mathbf{k}^2 + \omega_n^2 + 4\pi \Pi^t(\mathbf{k}, \omega_n)}. \tag{46}$$

$$D^l(\mathbf{k}, \omega_n) = \frac{4\pi}{\omega_n^2 + 4\pi \Pi^l(\mathbf{k}, \omega_n)}. \tag{47}$$

On the other hand, an isolated quasi-neutral plasma may be considered as a uniform isotropic material medium being characterized by the dielectric constant $\varepsilon = \varepsilon(\mathbf{x} - \mathbf{x}', t - t')$ and magnetic permeability $\mu = \mu(\mathbf{x} - \mathbf{x}', t - t')$. Heisenberg operators of macroscopic electromagnetic field satisfy Maxwell equations. If we were to designate the Heisenberg operator of a four-dimensional vector potential of electromagnetic field in medium through $A^c(x) = A^c(\mathbf{x}, t)$, then with the gauge-transformation $A_4^c \equiv 0$ we shall obtain the following equation for A^c in a momentum representation:

$$\left[\left(\omega^2 \varepsilon(\mathbf{k}, \omega) - \frac{\mathbf{k}^2}{\mu(\mathbf{k}, \omega)}\right)\delta_{ik} + \frac{k_i k_k}{\mu(\mathbf{k}, \omega)}\right] A_k^c(\mathbf{k}, \omega) = 0. \tag{48}$$

We shall determine Green's delay function of electromagnetic field in medium by the relationship

$$D_{\mu\nu}^R(x_1 - x_2) = \begin{cases} -i\,\mathrm{Sp}\{e^{(F - H_n)\beta}[A_\mu^c(x_1) A_\nu^c(x_2) - \\ \quad - A_\nu^c(x_2) A_\mu^c(x_1)]\}, & t_1 > t_2, \\ 0, & t_1 < t_2. \end{cases}$$

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where H_M is the Hamiltonian of the plasma in a macroscopic description, and F is the free energy of the plasma. Using equation (48) and the relationship of the commutation for the vector potential, there readily is obtained equation for Green's delay function:

$$\left[\left(\omega^2\varepsilon(\mathbf{k},\omega)-\frac{k^2}{\mu(\mathbf{k},\omega)}\right)\delta_{ik}+\frac{k_ik_k}{\mu(\mathbf{k},\omega)}\right]D^R_{kl}(\mathbf{k},\omega)=\delta_{il},$$
$$D^R_{4l}=D^R_{l4}=D^R_{44}=0,$$

solution of which has the form

$$D^R_{ik}(\mathbf{k},\omega)=\left(\delta_{ik}-\frac{k_ik_k}{k^2}\right)D^t(\mathbf{k},\omega)+\frac{k_ik_k}{k^2}D^l(\mathbf{k},\omega), \tag{49}$$

$$D^t(\mathbf{k},\omega)=\frac{4\pi\mu(\mathbf{k},\omega)}{\omega^2\varepsilon(\mathbf{k},\omega)\mu(\mathbf{k},\omega)-k^2},\quad D^l(\mathbf{k},\omega)=\frac{4\pi}{\omega^2\varepsilon(\mathbf{k},\omega)}. \tag{50}$$

As is known (see, for example, work [5]), the Fourier component of Green's delay function $D^R_{\mu\nu}(\mathbf{k},\omega)$ and the Fourier component of Green's temperature function in a medium (which is nothing else but $D_{\mu\nu}(\mathbf{k},\omega_n)$ (33)) are connected by the relationship

$$D^R_{\mu\nu}(\mathbf{k},i\omega_n)=-D_{\mu\nu}(\mathbf{k},\omega_n).$$

In other words, $D^R_{\mu\nu}(\mathbf{k},\omega)$ is the analytic continuation of the function $D_{\mu\nu}(\mathbf{k},-i\omega)$ to the upper half-plane of the complex variable ω

$$D^R_{\mu\nu}(\mathbf{k},\omega)=-D_{\mu\nu}(\mathbf{k},-i\omega). \tag{51}$$

Considering equality (51) and comparing relationships (45)-(47) and (49)-(50), we find an expression for dielectric and magnetic permeability of the plasma [9], valid at any values of ω, lying in upper half-plane of the variable ω

$$1-\varepsilon(\mathbf{k},\omega)=\frac{4\pi}{\omega^2}\Pi^l(\mathbf{k},-i\omega),$$

$$1-\frac{1}{\mu(\mathbf{k},\omega)}=\frac{4\pi}{k^2}\left[\Pi^t(\mathbf{k},-i\omega)-\Pi^l(\mathbf{k},-i\omega)\right],$$

or, according to relationships (42)-(43), we have

$$1-\varepsilon(\mathbf{k},\omega)=\frac{4\pi}{k^2}\Pi_{44}(\mathbf{k},-i\omega). \tag{52}$$

$$1-\frac{1}{\mu(\mathbf{k},\omega)}=\frac{2\pi}{k^2}\left[\frac{3\omega^2}{k^2}\Pi_{44}(\mathbf{k},-i\omega)-\Pi_{ll}(\mathbf{k},-i\omega)\right]. \tag{53}$$ 75

In distinction from macroscopic electrodynamics the magnitudes 76 $\varepsilon(\mathbf{k}, \omega)$ and $\mu(\mathbf{k}, \omega)$, determined by formulas (52)-(53), are valid at any values of $\mathbf{k}$ and ω, i.e., they are a known generalization of dielectric and magnetic permeability of macroscopic electrodynamics.

By means of a diagram technique the polarization operator $\Pi_{\mu\nu}(\mathbf{k}, \omega_n)$ in formulas (41)-(43) is calculated in any approximation and a formal replacement of $\omega_n \to -i\omega$ permits us to determine the dielectric and magnetic permeability according to formulas (52)-(53). In calculating $\Pi_{\mu\nu}$ usually there are discarded the terms proportional to higher degrees of e_λ^2. In distinction from quantum electrodynamics, where the parameter of the expansion is $e_\lambda^2/\hbar c$ (or $e_\lambda^2/\hbar v$ — v is the velocity of a particle), here, besides $e_\lambda^2/\hbar v \sim e_\lambda^2\hbar^{-1}m_\lambda^{1/2}\beta^{1/2}$, parameters of the expansion at a high temperature and weak shielding are provided by $\hbar^2 n^{2/3}\beta m_\lambda^{-1}$ and $e_\lambda^2 n^{1/2}\beta$ (and also their product $e_\lambda^2 n\beta^2\hbar^2 m_\lambda^{-1}$) and at low temperature and high density $-e_\lambda^2 m_\lambda/\hbar^2 n^{1/3}$. Thus, in disregarding terms of higher order in e_λ^2, we make an expansion in the indicated parameters depending upon conditions of problem. We readily are convinced of this if by means of the diagram technique we analyze the graphs of terms proportional to different degrees of e_λ^2.

For a nonrelativistic plasma $\beta^{-1} \ll m$ (where m is the mass of the electron), the magnetic permeability is practically equal to unity, and transverse and longitudinal parts of the polarization operator are identical. In this case polarization operator (34) has the form

$$\Pi_{lk}(\mathbf{k},\omega_n)=-\frac{\omega_n^2}{k^2}\delta_{lk}\Pi_{44}(\mathbf{k},\omega_n). \tag{54}$$

$$\Pi_{4l}(\mathbf{k}, \omega_n) = \Pi_{l4}(\mathbf{k}, \omega_n) - \frac{i\omega_n k_l}{\mathbf{k}^2} \Pi_{44}(\mathbf{k}, \omega_n), \qquad (55)$$

$$\Pi^l(\mathbf{k}, \omega_n) - \Pi^t(\mathbf{k}, \omega_n) = -\frac{\omega_n^2}{\mathbf{k}^2} \Pi_{44}(\mathbf{k}, \omega_n).$$

Energy Losses

The energy being lost per unit of time by particle flying through a plasma is equal to

$$-\frac{d\varepsilon_p}{dt} = \int \frac{\omega e'^2}{4\pi^3(1 - e^{-\omega\beta})} T_{\mu\mu'} D^F_{0\mu'\nu'}(\mathbf{q}, \omega) D^F_{0\mu\nu}(\mathbf{q}, \omega) \times$$
$$\times \operatorname{Im} K_{\nu\nu'}(\mathbf{q}, -i\omega)\, d^3q; \qquad (56)$$
$$\omega = \varepsilon_p - \varepsilon_{p-q}, \quad \varepsilon_p = \sqrt{\mathbf{p}^2 + M^2}.$$

It must be noted that during scattering of particle there takes place the relationship $\omega^2 \neq q^2$. Equality $\omega^2 = q^2$ would signify that flying particle radiated a photon, but in the approximation being considered (17) this does not occur.. Therefore $D^F_{0\mu\nu}(q, \omega)$ is a real function. Furthermore, in an arbitrary gauge transformation of the potentials it is possible in relationship (56) to make the substitution (see works[5, 2]).

$$D^F_{0\mu\nu}(\mathbf{q}, \omega) D^F_{0\mu'\nu'}(\mathbf{q}, \omega) = D_{0\mu\nu}(\mathbf{q}, -i\omega) D_{0\mu'\nu'}(\mathbf{q}, -i\omega).$$

Since, in relationship (56), $\operatorname{Im} D_{0\mu\nu}(q, -i\omega) = 0$, then, according to equations (31) and (33), we shall obtain

$$-\frac{d\varepsilon_p}{dt} = \frac{e'^2}{4\pi^3} \int \frac{\omega T_{\mu\nu}}{1 - e^{-\omega\beta}} \operatorname{Im} D_{\nu\mu}(\mathbf{q}, -i\omega)\, d^3q. \qquad (57)$$

In such a form formula (57) is applicable for investigating the energy losses of particle passing through any medium. In this case function $D_{\mu\nu}(x - x')$ is Green's thermodynamic function of the electromagnetic field in a medium which is expressed in terms of the dielectric and magnetic permeability of the medium according to formulas (45)-(47) and (52)-(53) (see Appendix 2).

Formula (57) is valid at any temperature of the plasma and

arbitrary velocities of a flying particle. However, at first we shall examine a nonrelativistic particle $v \ll 1$, passing through nonrelativistic plasma $\beta^{-1} \ll m$, where m is the mass of the electron. As a result, we shall obtain formulas which generalize the well known work of A. Larkin [1] in the case of multicomponent plasma

$$-\frac{d\varepsilon}{dt} = \int_0^\infty dq \int_{-1}^{1} dx \, \frac{8e'^2\omega}{e^{-\omega\beta}-1} \, \mathrm{Im} \frac{\Pi^1_{44}(\mathbf{q}, -i\omega)}{q^2 - 4\pi \Pi^1_{44}(\mathbf{q}, -i\omega)}, \tag{58}$$

$$\Pi^1_{44}(\mathbf{q}, -i\omega) = \sum_\lambda \frac{2e_\lambda^2}{(2\pi)^3} \int \frac{n^\lambda_{\mathbf{p}+\mathbf{q}/2} - n^\lambda_{\mathbf{p}-\mathbf{q}/2}}{\mathbf{pq}/m_\lambda - \omega} \, d^3p, \tag{59}$$

$$n^\lambda_{\mathbf{p}} = \left(e^{(p^2/2m_\lambda - \mu'_\lambda)\beta} + 1\right)^{-1}, \tag{60}$$

where $\omega = q(vx - q/2M)$, $x = \mathbf{pq}/pq$ $\mu'_\lambda = \mu_\lambda - m_\lambda$, and the polarization operator (34) is taken in the first approximation different from zero. Relationships (58)-(60) include the distribution function $n^\lambda_{\mathbf{p}}$ of only the fermions, since the distribution function of anti-fermions within a nonrelativistic limit vanishes. 77

We investigate in detail the region relative to high temperatures 78 and low density of the plasma when distribution function (60) coincides with the Boltzmann distribution. In this case imaginary part Π^1_{44} is equal to

$$\mathrm{Im}\,\Pi^1_{44}(\mathbf{q}, -i\omega) = \sum_\lambda n_\lambda e^2_\lambda \frac{\sqrt{2\pi m_\lambda \beta}}{2q} \times$$

$$\times \left(e^{-\omega\beta} - 1\right) e^{-\frac{m_\lambda \beta}{2}\left(\frac{\omega}{q} - \frac{q}{2m_\lambda}\right)^2},$$

where n_λ is the density of fermions of level λ

$$n_\lambda = \frac{2}{(2\pi)^3} \int n^\lambda_{\mathbf{p}} \, d^3p.$$

The method of calculating the integral (58) in a nonrelativistic region in detail is described in the indicated work [1], therefore

we shall give the final result for one of the most interesting cases. If the velocity v of a flying particle is great in comparison to the mean-thermal velocity $1/\sqrt{m_\lambda\beta}$ of the particles of plasma, then the energy losses are given by the formula*

$$-\frac{d\varepsilon_p}{dt} = \sum_\lambda \frac{4\pi e'^2 e_\lambda^2 n_\lambda}{m_\lambda v} \ln \frac{2Mm_\lambda v^2}{(M+m_\lambda)\hbar\omega_0}. \tag{61}$$

where

$$\omega_0^2 = \sum_\lambda \frac{4\pi n_\lambda e_\lambda^2}{m_\lambda}.$$

In formula (61) terms of an order $1/m_\lambda \beta v^2$ are discarded. As is evident from formula (61), the energy losses occur chiefly in an electron gas. Losses in ions must be taken into consideration only in the case when quasi-neutral plasma contains a high percentage of negative ions. Energy losses in the electron gas are calculated in work [1].

We shall consider further a relativistic particle being decelerated in nonrelativistic plasma $\beta^{-1} \ll m$. For this part of the integral of (57), in which the transmitted momentum q has nonrelativistic values, it is possible to use relationships (54)-(55), where

$$T_{\mu\nu} D_{\nu\mu}(\mathbf{q}, -i\omega) = \left(v^2 - \frac{1}{\varepsilon}\right)\frac{4\pi}{q^2-\omega^2\varepsilon}, \quad \omega = \mathbf{q}\mathbf{v},$$

where $\varepsilon = \varepsilon(\mathbf{q}, \omega)$ is given by formula (52). In particular, energy losses with transmission of a small momentum, included in the interval $0 \leq q \leq q_0$, where $q_0^2 \ll m/\beta$, have the form

$$-\frac{d\varepsilon_p}{dt} = \sum_\lambda \frac{4\pi e'^2 e_\lambda^2 n_\lambda}{m_\lambda v} \ln \frac{q_0 v}{\omega_0}.$$

In calculating the other part of the integral in (57), in which q acquires relativistic values, there must be taken into account, in general, both the relativistic and also the quantum effects.

In the most simple case of deceleration of a heavy relativistic

*All final formulas contain, in explicit form, the $\hbar$ and c.

particle only in an electron gas of a plasma, quantum effects are immaterial and energy losses from pair collisions are calculated by the ordinary (classical) procedure. Finally, the energy being lost per unit of time from a heavy relativistic particle in an electron gas of plasma is equal to

$$-\frac{d\varepsilon_p}{dt} = \frac{2\pi n e^2 e'^2}{mv}\left(\ln\frac{m^2v^4}{\pi n \hbar^2 e'^2\left(1-\frac{v^2}{c^2}\right)} - \frac{v^2}{c^2}\right).$$

where e, m and n are, respectively, the charge, mass, and density of electrons of plasma.

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Appendix 1

We shall use a certain general relationship, which may be useful also in other problems. For this purpose we shall consider Green's thermodynamic functions $G'_\lambda(x, x')$ of plasma particles in the presence of external current J(x) (see, for instance, [2])

$$G'_\lambda(x, x') = \langle T_\tau[\Psi_\lambda(x)\overline{\Psi}_\lambda(x')S']\rangle / \langle S'\rangle, \qquad (1.1)$$

$$S' = T_\tau e^{-\int I(x)A(x)d^4x}, \quad I(x) = j(x) + J(x).$$

where for arbitrary magnitude B there is adopted the designation

$$\langle B\rangle = \frac{\mathrm{Sp}\, e^{\left[\sum\limits_\lambda(\mu_\lambda N_\lambda - H_\lambda) - H_\gamma\right]\beta} B}{\mathrm{Sp}\, e^{\left[\sum\limits_\lambda(\mu_\lambda N_\lambda - H_\lambda) - H_\gamma\right]\beta}}.$$

Here $d^4x = d^3x\, d\tau$, where the integration over $\mathbf{x}$ is made within limits of the volume of plasma, whereas integral in the variable τ is taken from 0 to β. The dependence of the field operators on the variable τ is determined in the following way ("interaction picture"):

$$\Psi_\lambda(x) = e^{-(\mu_\lambda N_\lambda - H_\lambda)\tau}\psi_\lambda(\mathbf{x})e^{(\mu_\lambda N_\lambda - H_\lambda)\tau}.$$

At J(x) = 0 function (1.1) coincides with Green's thermodynamic function G_λ (32). According to the well known theorem connecting

N- and T-products of operators [10-11], we shall rewrite Green's function (1.1) in the form

$$G'_{\lambda}(x, x') = \langle N\,(e^{\Delta}\, e^{\Sigma}\, \Psi_{\lambda}(x)\, \bar{\Psi}_{\lambda}(x')\, \sigma] \rangle / \langle S' \rangle. \tag{1.2}$$

where

$$\sigma = e^{-\int J(x) A(x)\, d^4x},$$

$$\Sigma = \int d^4x\, d^4y\, G_{0\,\lambda\alpha\beta}(x-y) \frac{\delta^2}{\delta\bar{\Psi}_{\mu\lambda}(y)\, \delta\Psi_{\lambda\alpha}(x)},$$

$$\Delta = \frac{1}{2} \int d^4x\, d^4y\, D_{0\,\mu\nu}(x-y) \frac{\delta^2}{\delta A_{\nu}(y)\, \delta A_{\mu}(x)}.$$

By commutating the operators exp Δ and σ in formula (1.2), we shall obtain

$$G'_{\lambda}(x, x') = \langle D\,[e^{\Sigma}\, \Psi_{\lambda}(x)\, \bar{\Psi}_{\lambda}(x') \times$$
$$\times\, e^{-\frac{1}{2}\int J_{\mu}(x) D_{0\,\mu\nu}(x-y) J_{\nu}(y)\, d^4x\, d^4y}\, \sigma] \rangle / \langle S' \rangle. \tag{1.3}$$

The polarization operator (34) in coordinate representation is determined by the relationship [2]

$$\int \Pi_{\mu\nu'}(x-y)\, D_{\nu'\nu}(y-x)\, d^4y = \sum_{\lambda} e_{\lambda}\, \gamma_{\mu\alpha\beta} \frac{\delta G'_{\lambda\beta\alpha}(x, x)}{\delta J_{\nu}(x)} \bigg|_{J=0}. \tag{1.4}$$

Variational derivative with respect to external current in formula (1.4) is readily found if we use formula (1.3). As a result we shall have

$$\int \Pi_{\mu\nu'}(x-y)\, D_{\nu'\nu}(y-x)\, d^4y = -$$
$$- \int K_{\mu\nu'}(x-y)\, D_{0\,\nu'\nu}(y-x)\, d^4y, \tag{1.5}$$

where the correlation function $K_{\mu\nu}(x - y)$ is determined by formula (22). In relationship (1.5), by completing the Fourier transformation (26) we obtain formula (31). If we use the diagram technique, then formula (1.5) may be obtained also without involving variational derivatives.

Appendix 2

We shall apply methods of quantum field theory in statistical

physics to the problem on the deceleration of a charged particle in a substance. As is known, a charged particle flying through a substance loses its own energy owing to excitation and ionization of atoms of substance. Furthermore, the energy of flying particle is expended in surmounting the deceleration force, developing as a result of polarization of the medium by a charge of the particle. From the macroscopic point of view the energy losses of particles are considered to be a result of the excitation in medium of electromagnetic waves, which attenuate if medium has a complex dielectric permeability. Thus, the moving particle transmits energy to medium by means of electromagnetic field. Energy losses of charged particle in a substance can be calculated by methods of quantum field theory applied in many-body problems. The calculation made below is of well-known interest in methodology in view of future applications of the present method.

The Hamiltonian of the considered system in the Schrödinger picture will be written in the form of a sum

$$H_c + H_0' + H_1'.$$

Here H_c includes the Hamiltonian of particles of the medium, the Hamiltonian of free electromagnetic field H_γ (3), and energy of interaction of particles of medium with free electromagnetic field. H_0' is the Hamiltonian of external flying particle (fermion), and H_1' is energy of interaction of flying particle with the electromagnetic field

$$H_1' = \int j(\mathbf{x}) A(\mathbf{x}) d^3x.$$

where $j(\mathbf{x})$ is the operator of the four-dimensional current of the flying particle, and $A(\mathbf{x})$ is the operator (7) of the four-dimensional potential of free electromagnetic field.

The S_c-matrix of the quantum-mechanical system being considered is determined by the equation

$$i\frac{\partial S_c}{\partial t} = (H_c + H_0' + H_1')S_c.$$

By transformation

$$S_c = e^{-i(H_c + H_0')t}s_c,$$

we shall introduce the s_c-matrix describing the scattering of flying particle in the medium:

$$s_c = Te^{-i\int H_1'(x)d^4x}, \quad H_1'(x) = j(x)A(x), \tag{2.1}$$
$$j'(x) = e^{iH_0't}j'(x)e^{-iH_0't}, \quad A(x) = e^{iH_ct}A(x)e^{-iH_ct}.$$

By means of the s_c-matrix (2.1) we shall find statistically the averaged energy being lost by a flying particle per unit of time:

$$-\frac{d\varepsilon_p}{dt} = \frac{e'^2}{4\pi^3}\int \omega T_{\mu\nu}\Phi'_{\nu\mu}(\mathbf{q}, \omega)d^4q,$$

$$\Phi'_{\nu\mu}(\mathbf{q}, \omega) = (2\pi)^3\sum_{n,m} e^{(F-E_n)\beta}[A_\nu(0)]_{nm}[A_\mu(0)]_{mn}\delta(\mathbf{q}-\mathbf{p})_{mn}\delta(\omega-\omega_{mn}).$$

where $A(0)$ is the operator of the electromagnetic field in the Schrödinger picture taken at $\mathbf{x} = 0$; F is the free energy of the substance; and the remaining magnitudes were determined above.

The function $\Phi'_{\nu\mu}$ is directly connected with Green's thermodynamic function $D_{\nu\mu}$ of the electromagnetic field in the medium

$$D_{\nu\mu}(\mathbf{x}-\mathbf{x}', \tau-\tau') = Sp\{e^{(F-H_c)\beta}T_\tau[\mathcal{A}_\nu(\mathbf{x}, \tau)\mathcal{A}_\mu(\mathbf{x}', \tau')]\}, \tag{2.2}$$
$$\mathcal{A}(\mathbf{x}, \tau) = e^{H_c\tau}A(\mathbf{x})e^{-H_c\tau},$$

where τ and τ' vary within the interval from 0 to β.

Repeating the discussions which resulted in formula (30), we readily find

$$\Phi'_{\nu\mu}(\mathbf{q}, \omega) = \frac{\operatorname{Im} D_{\nu\mu}(\mathbf{q}, i\omega)}{\pi(1-e^{-\omega\beta})}.$$

Here $D_{\nu\mu}(\mathbf{q}, -i\omega)$ is obtained from $D_{\nu\mu}(\mathbf{q}, \omega_n)$ by the replacement $\omega_n \to -i\omega$ where $D_{\nu\mu}(\mathbf{q}, \omega_n)$ is the Fourier component of function (2.2).[8]

Thus, the energy lost by an external particle per unit of time during passage through a substance is equal to

$$-\frac{d\varepsilon_p}{dt} = \frac{e'^2}{4\pi^3}\int \frac{\omega T_{\mu\nu}}{e^{-\omega\beta}-1}\,\mathrm{Im}\,D_{\nu\mu}(\mathbf{q}, -i\omega)\,d^4q,$$

which coincides with formula (57).

If the medium being considered is a plasma, then, as was indicated above, function $D_{\nu\mu}(\mathbf{q}, \omega_n)$ can be directly calculated in any approximation. In a general case the material medium is characterized by dielectric permeability $\varepsilon = \varepsilon(\mathbf{x} - \mathbf{x}', t - t')$ and magnetic permeability $\mu = \mu(\mathbf{x} - \mathbf{x}', t - t')$ (for the sake of simplicity we assume the medium homogeneous and isotropic). By the usual method there are introduced operators of macroscopic electromagnetic field in the medium and Green's delay function $D^R_{\nu\mu}(\mathbf{x} - \mathbf{x}', t - t')$, whose Fourier component $D^R_{\nu\mu}(\mathbf{q}, \omega)$ is connected with $D_{\nu\mu}(\mathbf{q}, -i\omega)$ by relationship (51) where function $D^R_{\nu\mu}(\mathbf{q}, \omega)$ explicitly is determined by specifying $\varepsilon(\mathbf{q}, \omega)$ and $\mu(\mathbf{q}, \omega)$ of the medium being considered.

We now consider the energy losses of relativistic particle in a substance with transmission of a small momentum $q \ll \varepsilon_p$. The spatial dispersion $\varepsilon = \varepsilon(\omega)$ is ignored and we assume $\mu = 1$. Then we shall obtain

$$-\frac{d\varepsilon_p}{dt} = \frac{2e'^2 v}{\pi}\int_0^{q_0} dk\,k\,\mathrm{Im}\int \frac{(1 - v^2\varepsilon)\,\omega\,d\omega}{\varepsilon\,[k^2v^2 + \omega^2(1 - v^2\varepsilon)]\,(e^{-\omega\beta} - 1)}\,. \tag{2.3}$$

where v is the velocity of a particle, and q_0 is the upper limit of the transverse momentum being transmitted at which there still is valid the expression for $\varepsilon = \varepsilon(\omega)$.

In considering that real part of $\varepsilon(\omega)$ is an even function, and the imaginary part $\varepsilon(\omega)$ an odd function of ω, and considering the inequality

$$\frac{1}{e^{-\omega\beta} - 1} = -\frac{1}{2} - \frac{1}{2}\,\mathrm{Cth}\,\frac{\omega\beta}{2}\,.$$

formula (2.3) is readily transformed to the well-known relationship [12]

$$-\frac{d\varepsilon_p}{dt}=\frac{ie'^2 v}{\pi}\int dk k\int\frac{(1-v^2\varepsilon)\,\omega d\omega}{\varepsilon\left[k^2v^2+\omega^2(1-v^2\varepsilon)\right]}.$$

Literature

1. A. I. Larkin. Journal of experimental and theoretical physics, Vol. 37, 264 (1959).

2. A. I. Alekseyev. Advances in physical sciences, 73, 41 (1961).

3. G. C. Wick. Phys. Rev., 80, 268 (1950).

4. L. D. Landau. Journ. exper. and theoret. physics, 34, 262 (1958).

5. I. Ye. Dzyaloshinskiy and L. P. Pitayevskiy. Journ. exper. and theoret. physics, 36, 1797 (1959).

6. Ye. S. Fradkin. Journ. exper. and theoret. physics, 38, 157 (1960).

7. Ye. S. Fradkin. Nuclear Physics, 12, 465 (1959).

8. I. A. Akhiyezer and S. V. Peletminskiy. Journ. exper. and theoret. physics. 38, 1829 (1960).

9. I. A. Akhiyezer. Ukrainian physics journal. Vol. 6, 435 (1961).

10. T. Matsubura. Progress in Theoretical Physics. 14, 351 (1955).

11. J. L. Anderson. Physical Review, 94, 703 (1954).

12. L. D. Landau and Ye. M. Lifshits. Electrodynamics of solid media. Moscow, Fizmatgiz, 1955, p. 441.

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The Application of Quantum-Field
Theory Methods to Multibody
Problems. Moscow, Gosatomiz-
dat, 1963
Pages: 83-93

RADIATION OF A HIGH-TEMPERATURE PLASMA

A. I. Alekseyev and M. A. Troitskiy

Methods of quantum field theory in statistical physics are applied to problem on the radiation of a high-temperature plasma. There is found spectral decomposition of intensity of bremsstrahlung of a volume unit of plasma with the shielding of Coulomb field of ions taken into account. The production of a plasma by magnetic retardation also is investigated. The work was completed in 1961.

Bremsstrahlung of a high-temperature plasma, and also recombination and bright-line radiations of a low-temperature plasma were examined by a number of authors by means of classical methods (see, for example, [1, 2]). Of considerable interest is the application of new methods to the given problem in statistical physics, inasmuch as in a number of cases they give the most correct solution of the problem. Together with new results in certain limiting cases we obtain already well-known formulas. Nevertheless new solution is of definite methodical interest in view of the application of an analogous method to other noninvestigated problems. For this purpose there is examined the heat transfer of a high-temperature multicomponent plasma. The device discussed below is generalized readily for the region of low temperatures, in which the calculation of discrete levels of ion becomes essential and also for the case of the radiation of a magnetized plasma. For plasma, which is found in a constant

magnetic field there is obtained a general formula of the magnetic bremsstrahlung and certain limiting cases are investigated.

Bremsstrahlung of a High-Temperature Plasma

Hamiltonian H_0 of a quasi-neutral system of electrons and ions which are in thermal equilibrium have the following form in Schrödinger representation

$$H_0 = \sum_{\lambda} \int \psi^*_{\lambda}(\mathbf{x}) \frac{\mathbf{p}^2}{2m_{\lambda}} \psi_{\lambda}(\mathbf{x})\, d^3\mathbf{x} + \frac{1}{2} \sum_{\lambda, \lambda'} \frac{e_{\lambda} e'_{\lambda}}{|\mathbf{x}-\mathbf{x}'|} \psi^*_{\lambda}(\mathbf{x}) \psi^*_{\lambda'}(\mathbf{x}') \psi_{\lambda'}(\mathbf{x}') \psi_{\lambda}(\mathbf{x})\, d^3x\, d^3x',$$

where the sign λ numbers the particles of a given level. The operator of interaction H_1 of the plasma particles with the electromagnetic field*

$$H_1 = \sum_{\lambda} \left(-\frac{e_{\lambda}}{m_{\lambda}} \int \psi^*_{\lambda}(\mathbf{x}) \mathbf{A}\mathbf{p} \psi_{\lambda}(\mathbf{x})\, d^3x + \frac{e^2_{\lambda}}{2m_{\lambda}} \int \psi^*_{\lambda}(\mathbf{x}) \mathbf{A}^2 \psi_{\lambda}(\mathbf{x})\, d^3x \right)$$

results in bremsstrahlung and recombination and bright-line radiation of the plasma. Inasmuch as we are limited to the nonrelativistic region the basic contribution to the radiation will be introduced by electrons, the lightest particles. Therefore, from the entire sum of the operator H_1 subsequently we shall consider only the component pertaining to the electron with charge e and mass m.

As also in work [3], we shall determine the S-matrix, describing quantum-mechanical transitions of two quantum systems — plasma with Hamiltonian H_0 and free field of radiation with Hamiltonian H_{γ}

$$i \frac{\partial S}{\partial t} = (H_0 + H_{\gamma} + H_1) S.$$

---

*There is used such a gauge transformation of the electromagnetic potentials, at which scalar potential is equal identically to zero, and the vector potential **A** satisfies the condition div **A** = 0. Furthermore, it is assumed $\hbar = c = 1$.

By means of the transformation

$$S = e^{-i(H_0 + H_\gamma)t} s$$

we shall turn to the interaction picture, in which the s-matrix is defined as

$$i \frac{\partial s}{\partial t} H_1 s, \quad s = T e^{i\int H_1(\mathbf{x},\, t)\, d^3x\, dt};$$

$$H_1(\mathbf{x},\, t) = -\frac{e}{m} \psi^*(\mathbf{x},\, t)\mathbf{A}(\mathbf{x},\, t)\mathbf{p}\psi(\mathbf{x},\, t) +$$

$$+ \frac{e^2}{2m} \psi^*(\mathbf{x},\, t)\mathbf{A}^2(\mathbf{x},\, t)\psi(\mathbf{x},\, t);$$

$$\psi(\mathbf{x},\, t) = e^{iH_0 t}\psi(\mathbf{x})e^{-iH_0 t}, \quad A(\mathbf{x},\, t) = e^{iH_\gamma t}\mathbf{A}(\mathbf{x})e^{-iH_\gamma t},$$

where symbol T before the operators designates the T-product [4]. 84

Radiation of plasma in first approximation according to H_1 is 85
described by the following matrix element of the s-matrix

$$s_{mk,\, n0} = \frac{ie(2\pi)^4}{m} \sqrt{\frac{2\pi}{V\omega}}\, [\psi^*(0)\psi(0)]_{mn}\, \mathbf{1}^\Lambda \mathbf{p}_n \delta(\mathbf{k} + \mathbf{p}_{mn}) \times$$

$$\times \delta(\omega + \omega_{mn}); \tag{1}$$

$$\mathbf{p}_{mn} = \mathbf{p}_m - \mathbf{p}_n; \quad \omega_{mn} = E_m - E_n,$$

where $\psi(0)$ is the operator of electron field in the Schrödinger representation taken at $\mathbf{x} = 0$; 1^Λ and ω – respectively the vector of polarization and energy of photon with the momentum $\mathbf{k}$ ($\mathbf{k}^2 = \omega^2$), and $\mathbf{p}_n$ and E_n – respectively the total momentum and total energy of the plasma in the n-th state; V is the volume of the plasma.

Using formula (1), we find statistically the averaged energy dQ being radiated by a volume unit of plasma per unit of time in the interval $d\omega$

$$dQ = \frac{e^2\omega^3 d\omega}{2\pi m^2} \int \left(\delta_{jl} - \frac{k_j k_l}{\mathbf{k}^2}\right) \Phi_{lj}(\mathbf{k},\, \omega)\, d\Omega_{\mathbf{k}}; \tag{2}$$

$$\Phi_{lj}(\mathbf{k},\, \omega) = (2\pi)^3 \sum_{n,\, m} e^{(\Omega + \sum_\lambda \mu_\lambda N_\lambda - E_n)\beta} [\psi^*(0)\psi(0)]_{nm} \times$$

$$\times [\psi^*(0)\psi(0)]_{mn}\, \mathbf{p}_{nl}\mathbf{p}_{mj}\, \delta(\mathbf{k} + \mathbf{p}_{mn})\delta(\omega + \omega_{mn}).$$

where $\beta = 1/kT$, Ω is the thermodynamic potential of the plasma, and μ_λ and N_λ respectively are chemical potential and total number of particles of level λ. By the repeated vector subscripts i, j there is implied a summation from one to three.

By following the method presented in work [3], it is readily proven that function $\Phi_{ij}(\mathbf{k}, \omega)$ is connected with analytic continuation of Fourier-component $K_{ij}(\mathbf{k}, \omega_n)$ of the correlation function $K_{ij}(x_1 - x_2)$

$$K_{ij}(x_1 - x_2) = -\mathrm{Sp}\{e^{(\Omega + \sum_\lambda \mu_\lambda N_\lambda - H_0)\beta} T_\tau [(\tilde{\psi}^*(x_1) p_i \tilde{\psi}(x_1)) \times$$

$$\times (\tilde{\psi}^*(x_2) p_j \tilde{\psi}(x_2))]\},$$

$$\tilde{\psi}(x) = e^{-(\sum_\lambda \mu_\lambda N_\lambda - H_0)\tau} \psi(\mathbf{x}) e^{(\sum_\lambda \mu_\lambda N_\lambda - H_0)\tau},$$

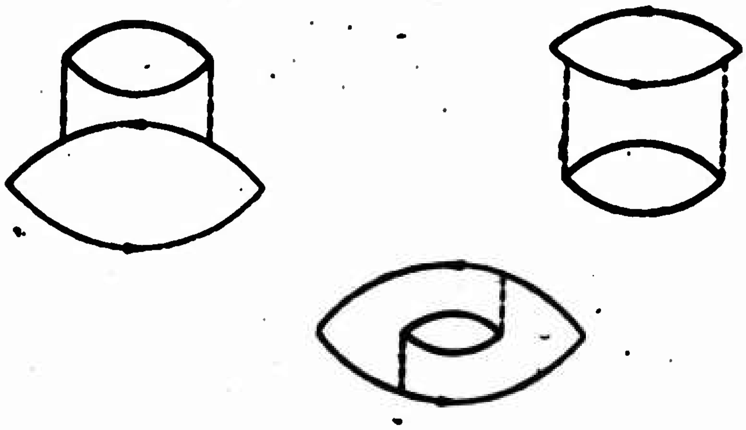
where p_i is the differential operator of the momentum $0 \leq \tau \leq \beta$ and T_τ designates the T-product, in which the order of operators occurs according to the variable τ.

The indicated relationship has the form

$$\Phi_{ij}(\mathbf{k}, \omega) = \frac{\mathrm{Im}\, K_{ij}(-\mathbf{k}, i\omega)}{\pi(1 - e^{\omega\beta})}. \qquad (3)$$

Formula (2) with function $\Phi_{ij}(\mathbf{k}, \omega)$ (3) is useful for investigating the bremsstrahlung and the recombination and bright-line radiations. Below there is considered only the bremsstrahlung.

In order to calculate $K_{ij}(\mathbf{k}, \omega_n)$ there is applied the well-known diagram technique (see for example, [5]) and then the formal replacement of $\omega_n \to i\omega$ and $\mathbf{k} \to -\mathbf{k}$ permits us to find $\Phi_{ij}(\mathbf{k}, \omega)$ by formula (3). In the case of a high-temperature plasma the first approximation different from zero for the imaginary part of function $K_{ij}(-\mathbf{k}, i\omega)$ is given by the graphs, on which the factor $(-4\pi)q^2$ corresponds to dashed line and to the solid thin line — Green's electron thermodynamic



function in the zero approximation (see Figure).

$$G_0(p, \omega_m) = \frac{-1}{i\omega_m + \mu - \frac{p^2}{2m}},$$

where μ is the chemical potential of the electron gas. In the figure the closed loop strung to the dashed line, belongs to ions of the plasma, since at first we consider the dipole radiation without taking into account shielding. Here the sum of graphs corresponding to a dipole radiation of electron on electron gives terms which mutually are reduced.

As is known, in the nonrelativistic region the momentum of a radiated photon is much less than the momentum of a radiating particle, and therefore in a dipole approximation the momentum of the photon $\mathbf{k}$ in $K_{ij}(-\mathbf{k}, i\omega)$ must be set equal to zero. Then integral over angles 86
in formula (2) may be calculated and dQ will be expressed through 87
$K_{ii}(0, i\omega)$, where

$$K_{ll}(0, \omega_n) = \frac{e^2}{2\pi^4\beta^2} \int \sum_{\omega_m \omega'_n} G_0(\mathbf{p}, \omega_m) G_0(\mathbf{p}-\mathbf{q}, \omega_m - \omega_{n'} - \omega_n) \times$$
$$\times \left[\mathbf{p}^2 G_0^2(\mathbf{p}, \omega_m - \omega_n) + (\mathbf{p}-\mathbf{q})^2 G_0^2(\mathbf{p}-\mathbf{q}, \omega_m - \omega_{n'}) + \right.$$
$$\left. + \mathbf{p}(\mathbf{p}-\mathbf{q}) G_0(\mathbf{p}, \omega_m - \omega_n) G_0(\mathbf{p}-\mathbf{q}, \omega_m - \omega_{n'}\right] \times$$
$$\times \Pi(q, \omega_{n'}) \frac{d^3q\, d^3p}{q^4}; \qquad (4)$$

$$\Pi(q, \omega_n) = \sum_\lambda{}' \frac{e_\lambda^2}{(2\pi)^3} \int \frac{n^\lambda_{\mathbf{p}+\mathbf{q}/2} - n^\lambda_{\mathbf{p}-\mathbf{q}/2}}{\frac{\mathbf{pq}}{m_\lambda} - i\omega'_n} d^3p. \qquad (5)$$

Here, $\omega_{n'} = \frac{2n'\pi}{\beta}$, $\omega_m = \frac{(2m+1)\pi}{\beta}$, where n', $m = 0, \pm 1, \pm 2, \ldots$.

Summation in formula (5) occurs in all levels of ions.

The ignored graphs, being proportional to the small dimensionless parameters $e_\lambda e m^{1/2}\beta^{1/2}\hbar^{-1} \ll 1$ and $e_\lambda e m^{-1}\beta^2 n\hbar^2 \ll 1$ (n is the density of electrons) give a contribution, considerably less than the basic term (4). The smallness of the parameter $e_\lambda e m^{1/2}\beta^{1/2}\hbar^{-1}$ permits us to take into account the interaction between electron and ion in the Born approximation, whereas in the realization of the inequality $e_\lambda e m^{-1}\beta^2 n\hbar \ll 1$ there may be disregarded the effects of the medium's polarization type.

Relationships (2)-(5) are applicable both for the Fermi, and also the Bose laws of the distribution of particles according to the momentum n_p^λ. However, below we shall assume that distribution function of electrons and ions of plasma coincides with the Boltzmann distribution.

Using the formulas:

$$\frac{1}{\beta}\sum_{\omega_n}\frac{1}{i\omega_n+z}=\frac{1}{2}\operatorname{cth}\frac{z\beta}{2},\quad \omega_n=2n\pi/\beta;$$

$$\frac{1}{\beta}\sum_{\omega_m}\frac{1}{i\omega_m+z}=\frac{1}{2}\operatorname{th}\frac{z\beta}{2},\quad \omega_m=(2m+1)\pi/\beta,$$

where z is an arbitrary complex variable it is readily shown that

$$\sum_{\omega_n}\frac{1}{(i\omega_n+z_1)(i\omega_n+z_2)(i\omega_n+z_3)}=\frac{1}{z_1 z_2 z_3}+f(z_1, z_2, z_3). \qquad (6)$$

Here z_1, z_2 and z_3 are complex variables, and the function $f(z_1, z_2, z_3)$ does not have poles with real values z_1, z_2 and z_3. If the summation in equality (6) is made in $\omega_n = (2n + 1)\pi/\beta$; then we shall obtain a function, which does not have poles with real values z_1, z_2, and z_3 (the first component in the right-hand side of equality (6) will be lacking).

Considering realtionship (6), we shall use the equality

$$G_0(\mathbf{p}, \omega_m) G_0(\mathbf{p}, \omega_m - \omega_n) = \frac{1}{i\omega_n}[G_0(\mathbf{p}, \omega_m - \omega_n) - G_0(\mathbf{p}, \omega_m)] \tag{7}$$

and will discard all components in expression (4) whose imaginary part of analytic continuation $i\omega_n \to -\omega$ is equal to zero. We shall obtain

$$K_{ll}(0, \omega_n) = \frac{-e^2}{2\pi^4\beta^2\omega_n^2} \int \sum_{\omega_m, \omega_{n'}} G_0(\mathbf{p}, \omega_m) G_0(\mathbf{p} - \mathbf{q}, \omega_m - \omega_{n'} - \omega_n) \Pi(\mathbf{q}, \omega_{n'}) \frac{d^3q\, d^3p}{q^2}. \tag{8}$$

The summation in formula (8) is made in an elementary manner. As a result for energy dQ being radiated by a volume unit of plasma per unit of time in the interval $d\omega$ we shall obtain in accordance with work [1] the following expression:*

$$dQ = \sum_\lambda \frac{16}{3} \frac{e_\lambda^2}{\hbar c} r_0^2 c^2 n n_\lambda \sqrt{\frac{2m\beta}{\pi}}\, e^{-\frac{\omega\beta}{2}} K_0\left(\frac{\omega\beta}{2}\right). \tag{9}$$

where $r_0 = \frac{e^2}{mc^2}$ is the classical electron radius, n and n_λ is the density respectively of the electrons and ions of level λ, and K_0 is the MacDonald function.

If the shielding of Coulomb interaction of particles of plasma is significant then the dashed line with the strung ionic loop (see figure) should be replaced by a dashed line on which in succession there is strung an infinite number of electron and ionic loops. Then to the graphs, shown in the figure, we must add still two graphs which are two electron loops with three tips, connected to each other by two dashed lines, on which there is strung a large number of electron and ionic loops.

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*All final formulas contain the constants $\hbar$ and c in explicit form.

In the most simple case of quiescent ions the Coulomb field of ions may be assumed to be an external field in which the electron gas moves. The discussed apparatus is readily generalized for this case, where for $K_{ii}(0, \omega_n)$, the shielding action of electrons taken into account, we shall obtain

$$K_{ll}(0, \omega_n)=\sum_{\lambda}\frac{e^2 e_\lambda^2 n_\lambda}{2\pi^4\omega_n^2}\int\sum_{\omega_m} G_0(p,\omega_m)G_0(p-q,\omega_m-\omega_n)\times \frac{q^2 d^3 q\, d^3 p}{(q^2+\varkappa^2)^2},$$

where $\varkappa^2 = 4\pi e^2 n\beta$ is the square of the inverse Debye length. With secured ions the energy dQ radiated by a volume unit of plasma per unit of time in the interval $d\omega$ has the following form:

$$dQ=\sum_{\lambda}\frac{8}{3}\frac{e_\lambda^2}{\hbar c} r_0^2 c^2 n n_\lambda\sqrt{\frac{2m\beta}{\pi}}\, e^{-\frac{\omega\beta}{2}} F(\omega, \varkappa^2)\, d\omega;$$
$$F(\omega, \varkappa^2)=\int_0^\infty e^{-\frac{\omega\beta}{4}\left(\frac{1}{y}+y\right)}\frac{y\,dy}{\left(y+\frac{\hbar^2\varkappa^2}{2m\omega}\right)^2}. \qquad (10)$$

For high frequencies $\omega\beta \gg 1$ the shielding may be disregarded, the function $F(\omega, 0)$ coincides with MacDonald's function

$$F(\omega, 0)=K_0\left(\frac{\omega\beta}{2}\right),$$

and formula (10) corroborates results of work [1].

At low frequencies the shielding becomes significant if there is fulfilled the condition

$$\omega\leq\sqrt{\frac{\hbar^2\varkappa^2\beta}{m}}\,\frac{1}{\beta}. \qquad (11)$$

In taking into account the shielding, function $F(\omega, \varkappa^2)$ at low frequencies (11) takes the following form:

$$F(\omega, \varkappa^2)=\ln\frac{8m}{\hbar^2\varkappa^2\beta}-C-1-\omega\frac{\hbar^2\varkappa^2\beta}{32m}\ln\frac{8m}{\hbar^2\varkappa^2\beta},$$

where $C = 0.577$ is the Euler constant.

For the Boltzmann distribution of particles by momentum the ratio

$$\frac{\hbar^2 \varkappa^2 \beta}{m} \sim \frac{e_\lambda e \beta^2 n \hbar^2}{m} \ll 1 .$$

is fulfilled automatically, if only there is fulfilled the condition of the Born approximation $e_\lambda e m^{1/2} \beta^{1/2} \hbar^{-1} \ll 1$. Thus, the effects of polarization of a medium are immaterial in Born case and the calculation for shielding is not reflected markedly in the magnitude of the total energy radiated by a volume unit of the plasma per unit of time. Calculation of the shielding of the Coulomb ion field alters the spectral decomposition of the radiation intensity only at very low frequencies (11) by removing logarithmic divergence characteristic for radiation in Coulomb field.

There is basis for expecting that in case of strong shielding $e_\lambda e \beta^2 n \hbar m^{-1} \gg 1$ formula (10) will remain accurate in order of magnitude. Then energy radiated by unit of volume of plasma in unit of time in order of magnitude will be equal

$$Q = \sum_\lambda \frac{2^5}{3^2} \frac{e^2_\lambda}{\hbar c} r_0^2 c^2 n n_\lambda \sqrt{\frac{2m}{\pi \beta}} \frac{m}{\hbar^2 \varkappa^2 \beta} ;$$

$$\frac{\hbar^2 \varkappa^2 \beta}{m} \gg 1.$$

Magnetic Bremsstrahlung

We now investigate magnetic bremsstrahlung of plasma in constant uniform magnetic field H_z, directed along axis oz, under the condition that the absorption may be ignored and, consequently, all radiation emerges from volume of plasma outwardly. Inasmuch as the major contribution to the magnetic bremsstrahlung is given by light particles, we shall write out in the Schrödinger notation the Hamiltonian only of the electron gas of the plasma

$$H = \int \Psi^* (\mathbf{x}) \frac{P_j^2}{2m} \Psi (\mathbf{x}) d^3 x,$$

$$P_1 = -i \frac{\partial}{\partial x_1} + e H_z x_2; \; P_2 = -i \frac{\partial}{\partial x_2}; \; P_3 = -i \frac{\partial}{\partial x_3} .$$

We shall assume the plasma fairly well heated, so that the condition of Born approximation is fulfilled and energy of interaction of electrons with electrons and ions may be ignored. Interaction of electrons with electromagnetic field

$$-\frac{e}{m}\int \Psi^*(x) A_j P_j \Psi(x)\, d^3x + \frac{e^2}{2m}\int \Psi^*(x) A_j^2 \Psi(x)\, d^3x$$

results in a magnetic bremsstrahlung. Just as this was done above, we shall determine the s-matrix describing the radiation process. Then in a dipole approximation the magnetic bremsstrahlung is described by matrix element of the s-matrix

$$s_{mk,\,n0} = i\frac{e}{m}\sqrt{\frac{2\pi}{V\omega}}\, l_j^{\lambda}\int (\Psi^*(x)\, e^{-ikx} P_j \Psi(x))_{mn}\, d^3x \times$$
$$\times 2\pi\delta(E_m - E_n + \omega),$$

where E_n is the total energy of electron gas of plasma in the n-th state.

The statistically averaged energy Q, being emitted by a volume unit of plasma per unit of time has the form

$$Q = \frac{e^2}{2\pi m^2}\int\left(\delta_{ji} - \frac{k_j k_i}{k^2}\right)\Phi'_{ij}(\mathbf{k}, \omega)\,\omega^2\, d\omega\, d\Omega_k\,;$$

$$\Phi'_{ij}(\mathbf{k}, \omega) = \frac{1}{V}\int \sum_{n,\,m} e^{(\Omega + \mu N - E_n)\beta}\,[\Psi^*(x)\, e^{ikx} P_i \Psi(x)]_{nm} \times$$

$$\times [\Psi^*(x')\, e^{-ikx'} P'_j \Psi(x')]_{mn}\,\delta(E_m - E_n + \omega)\, d^3x\, d^3x'.$$

Here Ω and μ are respectively the thermodynamic and chemical potentials of the electron gas and N is the total number of electrons in a volume V.

In a dipole approximation in function $\Phi'_{ij}(\mathbf{k}, \omega)$ it is possible to set $\mathbf{k} = 0$. Then we shall obtain

$$Q = \frac{4e^2}{3m^2}\int_0^\infty [\Phi'_{11}(0, \omega) + \Phi'_{22}(0, \omega)]\,\omega^2\, d\omega +$$

$$+ \frac{4e^2}{3m^2}\int_0^\infty \Phi'_{33}(0, \omega)\,\omega^2\, d\omega. \qquad (12)$$

the latter component in formula (12) does not make a contribution in the considered approximation, therefore subsequently it will be discarded. By virtue of axial symmetry of problem, functions $\Phi'_{11}(0, \omega)$ and $\Phi'_{22}(0, \omega)$ are equal to each other. Thus, in order to calculate Q it is sufficient to know one of the named functions (for instance, Φ'_{22}).

If we introduce the correlation function

$$K_{ij}(\tau-\tau') = \frac{1}{V}\int \mathrm{Sp}\{e^{(\Omega+\mu N-H)\beta}\, T_\tau\, [\Psi^*(x)P_i\Psi(x)\times \\ \times(\Psi^*(x')P'_j\Psi(x'))]\}\, d^3x\, d^3x'; \\ \Psi(x) = e^{-(\mu N-H)\tau}\,\Psi(x)\, e^{(\mu N-H)\tau}, \tag{13}$$

there is readily proven the equality

$$\Phi'_{ij}(0, \omega) = \frac{\mathrm{Im}\, K_{ij}(i\omega)}{\pi(1-e^{\omega\beta})},$$

where the function $K_{ij}(i\omega)$ is obtained from the Fourier component $K_{ij}(\omega_n)$ of function (13) by replacing $\omega_n \to i\omega$. Using the diagram technique there is readily found

$$K_{ij}(\omega_n) = -\frac{1}{V\beta}\sum_{\omega_m}\int P_i\, G_0(\mathbf{x}, \mathbf{x}'; \omega_m)\, P'_j \times \\ \times G_0(\mathbf{x}', \mathbf{x}; \omega_m-\omega_n)\, d^3x\, d^3x'; \\ \omega_n = 2n\pi/\beta;\ \omega_m = (2m+1)\pi/\beta,$$

where $G_0(\mathbf{x}, \mathbf{x}'; \tau-\tau')$ is Green's thermodynamic function of electron in a magnetic field which is investigated in detail in work [6]. As a result of simple calculations for electron gas with Boltzmann's law of distribution of particles based on momentum we shall obtain

$$\Phi'_{22}(0, \omega) = \frac{1}{2}\,\frac{nm\,\omega_H}{e^{\omega\beta}-1}\,\delta(\omega-\omega_H),$$

where n is the density of electrons, and $\omega_H = eH_z/mc$ is Larmor frequency of electron in a magnetic field. As we might have expected the radiation occurs only at the frequency ω_H.

Finally the energy radiated by a volume unit of a magnetized plasma per unit of time is equal to

$$Q=\frac{4e^2\omega_H^2\hbar}{3mc^3}\,\frac{n}{e^{\hbar\omega_H\beta}-1}. \qquad (14)$$

As can be seen from formula (14), magnetic bremsstrahlung exponentially attenuates with a decrease in temperature. However, the temperature of the plasma must remain fairly high so that condition of Born approximation is fulfilled.

Formula (14) is obtained by considering the fact that the motion of an electron in a magnetic field obeys laws of quantum mechanics. In classical limit $\hbar \to 0$ formula (14) changes to the well-known result [7]

$$Q=\frac{4e^2\omega_H^2 n}{3mc^3\beta}. \qquad (15)$$

Thus, the radiation of a magnetized plasma is described by means of classical theory (15) only in the case when energy of radiated quantum $\hbar\omega_H$ is much lower than the mean thermal energy of an electron ($\hbar\omega_H \ll 1/\beta$), which is realizable in fairly weak magnetic fields and at a high electron gas temperature. In the converse case ($\hbar\omega_H \gtrsim 1/\beta$) there must be taken into account the quantum character of motion of electron in the magnetic field; here the radiation of a magnetized plasma is given by formula (14).

Literature

1. V. I. Kogan and A. B. Migdal. Dependence of spectrum of bremsstrahlung on the electron temperature of a plasma. In the collection: "Plasma physics and the problem of controlled thermonuclear reactions." V. I, Moscow, Publishing House of the Academy of Sciences of the USSR, 1958, pp. 172-177.

2. V. I. Kogan. Recombination radiation of a hydrogen plasma. In the collection: "Plasma physics and the problem of controlled thermonuclear reactions." V. III, Moscow, Publishing House of Academy of Sciences of the USSR, 1958, pp. 99-103.

3. A. I. Alekseyev. Deceleration of a relativistic particle in a plasma. (See present collection, p. 65.)

4. G. C. Wick. Physical Review, 80, 268 (1950).

5. A. I. Alekseyev. "Advances in physical sciences." 73, 41 (1961).

6. I. A. Akhiyezer. "Journ. exper. and theoret. physics," 40, 954 (1961).

7. B. A. Trubnikov and A. Ye. Bazhanov. Magnetic radiation of a layer of plasma. In the collection: "Plasma physics and the problem of controlled thermonuclear reactions." V. III, Moscow, Publishing House of Academy of Sciences of the USSR, 1958, pp. 121-147. 83

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CTP	(1)	
TDDEI		1
TDDF		1
TDES		2
ESG/K	(1)	
ESW/T1	(1)	
TDET		2
ETT4/D	(1)	
[illegible]/F	(1)	
TDET (PGE)		5
PAB	(1)	
PAE	(1)	
PCA	(1)	
[illegible]	(1)	
PLA	(1)	
TDPS		1
Det #3 (FTD)		1
Det #4 (FTD)		1

Organization		Nr. Cys.
OTHER DOD AGENCIES		
DDC		20
U.S. Army (FSTC)		9
BSO	(1)	
GET	(1)	
NOTS China Lake		1
PAC Msl Range		1
OTHER GOVERNMENT AGENCIES		
AEC (Tenn)		2
AEC (Wash)		2
FAA (SS-10)		1
NAFEC		1
NASA (ATSS-T)		1
DISTRIBUTION TO BE MADE BY		
DIA (DIACO-3)		17
DIAAP-1C1	(1)	
DIAAP-1H2	(1)	
DIAST-A	(1)	
DIAST-5C	(1)	
U.S. Navy (ONR)	(1)	
OP922F2	(1)	
CIA (SD)	(5)	
NSA (CREF/CDB)	(6)	